UNIVERSITY OF MORATUWA
Faculty of Engineering
Department of Electronic & Telecommunication Engineering
B.Sc. Engineering
Mid-Semester Examination

EN2040 - Random Signals and Processes

Time Allowed: 1 hour — July 2015

Question 1

The table below shows the bivariate probability distribution of two random variables, X_1 and X_2.

	$X_1 = 1$	$X_1 = 2$	$X_1 = 3$
$X_2 = 1$	0.12	0.06	0.22
$X_2 = 2$	0.05	0.02	0.13
$X_2 = 3$	0.13	0.02	0.25

i) Calculate the marginal distributions of X_1 and X_2.

ii) Calculate the conditional distribution of X_1 given $X_2 = 3$.

iii) Compute $E(X_1)$, $E(X_2)$, $var(X_1)$, $E(X_1X_2)$ and $cov(X_1, X_2)$.

iv) Briefly explain whether the two random variables are correlated and/or independent.

[25 marks]

Question 2

A fair coin is tossed n times. The variable X_i describes the outcome of the i-th toss: $X_i = 0$ if Heads and $X_i = 1$ if Tails. Let $X = \sum_{i=1}^{n} X_i$. Provide an expression for the probability distribution of X.

[15 marks]

Question 3

A mobile phone manufacturer in China makes both good (G) and bad (B) mobile phones. The lifetime in seconds of G and B phones are characterized by their CDFs $F_G(t)$ and $F_B(t)$, respectively. The probability of a randomly selected phone being G is p.

i) Find the probability that a randomly selected phone still functions after T seconds.

ii) To eliminate the B phones, every phone is tested for T seconds. The phones that fail the test are sent to Sri Lanka, and the survivors are sent to the United States. For a target of 99% of the phones sent to the United States to be good, find an expression that relates T, p, F_G and F_B.

iii) If $F_G(t)$ and $F_B(t)$ are exponential distributions with rate λ and 1000λ, respectively, show that

$$T = \frac{1}{999\lambda} \ln\left(\frac{99(1-p)}{p}\right).$$

Hint: The CDF of an exponentially distributed random variable of rate μ is given by $F_\mu(x) = 1 - e^{-\mu x}$.

[30 marks]

Question 4

Consider a binary channel where messages $m = 0$ and $m = 1$ are transmitted using a negative and a positive pulse, respectively. The probability of transmitting a positive pulse is p_1 and the probability of transmitting a negative pulse is p_0. In the absence of noise, the received pulse corresponding to 1 is $p(t)$, and the received pulse corresponding to 0 is $-p(t)$. The peak amplitude of $p(t)$ is A_p at $t = T_p$. However, the channel is contaminated with zero mean Gaussian noise having a variance of σ_n^2. Thus, the received signal is given by $\pm p(t) + n(t)$. To detect the transmitted messages, each pulse is sampled at $t = T_p$, and the amplitude of the sample is compared with a decision threshold τ. Show that the optimum (error minimizing) threshold $\tau^\star$ is given by

$$\tau^\star = \frac{\sigma_n^2}{2A_p} \ln \frac{p_0}{p_1}.$$

Hint: $\frac{d}{dx}Q(x) = -\frac{1}{\sqrt{2\pi}}e^{\frac{-x^2}{2}}$ where $Q(x) = 1 - \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{\frac{-y^2}{2}} dy$.

[30 marks]